THE NEUTRINOS, THE GRAVITATION AND THE GAUGE FORMALISM

Valery Koryukin

Department of Applied Mathematics, Mari State Technical University
Box 179, Main Post–Office, Yoshkar–Ola, 424000, Russia
e-mail: koryukin@mpicnit.mari.su

The detection and the research of the neutrinos background of Universe are the attractive problems. This problems do not seem the unpromising one in the case of the high neutrinos density of Universe. It was offered before to use the low energy neutrinos background of Universe for the explanation of the gravitational phenomena with the quantum position attracting the Casimir's effect for this. As a result it was connected the gravitational constant with the parameters characterizing the electroweak interactions. If now we shall be based on the results of the experements fixing the equality of the gravitation mass and the inert one then it can consider that the spectrum of the particle masses is defined by their interaction with the neutrinos background of Universe. This statement is confirmed what the rest mass of the photon is equal to zero in contradistinction to the masses of the vector bosons W^+, W^-, Z^0 whiches interact with the neutrinos immediately.

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1. The Casimir's effect and the gravitation

The making of a physical theory embracing all an energy spectrum of interactions is a fairly difficult task. In consequence a construction of asymptotical theories both in high-energy and low-energy ends of this spectrum was justified historically. The most considerable success attended the work in the high-energy approximation, in a result of which was made quantum electrodynamics (QED) giving the prediction confirming experimentally with the remarkable precision. Naturally, that this theory became the imitation specimen by the construction of the analogous theories of the strong interaction (the quantum chromodynamics (QCD)) and the weak one (Salam-Weinberg model). The every possible theories describing continuum with the large number of particles such as the theories of a solid body, a liquid, a gas, a plasma, an electromagnetic radiation, shells, nuclei at low energies and also General Relativity (GR) is related to the opposite end of the energy spectrum. GR look the exclusion against this background, what gave the reason to consider the gravitation is only the effect of the existence of the space-time curvature.

Admittedly the theories wrecking the present idea are appearing in the second half of our century such as the two-tensor gravitation theory [1], in which was made the attempt to rewrite the theory of nuclear interactions into the geometrical language. It can attribute to like works also and the gauge theory of the dislocations and disclinations [2]. In consequence of this the transfer to the geometrization description as the most comfortable one in the long-wave length range for any interactions is the logical one. Thereby it is wrecked fully the exceptionality of the gravitation and the forces corresponding to it are not to be distinguished between others, such as Yukawa's forces and the Van der Waals' forces. So it is necessary to show that the gravitation interaction is not a fundamental one, but the one is induced by others interactions as possible hypothetical ones. The more so, that the gravitational constant $G_N \approx 6.7 \cdot 10^{-39}\ GeV^{-2}$ (it is used the system of units $\hbar = c = 1$, where $2\pi\hbar$ is the Planck's constant and c is the light speed) is a suspiciously small value and a dimensional one furthermore (as is known the latter prevent to the constraction of the renormalizable quantum theory).

Before building the theory of the induced gravitation on the base of the hypothetical interactions and the hypothetical particles it was necessary to verify the possibility of the utilization of the known particles and the known interactions for this purpose. Naturally that the neutrinos are the most suitable particles for this taking into account their penetrating ability, which allow them to interact with all the substance of the macroscopic body — not with the surface layer only. As is known [3], already in 30th Gamow and Teller offered to use the neutrinos for the explanation of the gravitation, but their mechanism provided the direct exchange of the pairs consisting of a neutrino and an antineutrino and therefore the one does not correspond to the modern conceptions of the theory of interactions.

Bashkin's works appearing in 80th on a propagation of the spin waves in the polarized gases [4] allowed to make the supposition [5], that the analogous collective oscillations are

possible under certain conditions as well as in the neutrinos medium. Since the collective oscillations can induce an interaction between particles, Bashkin's works make us to pay attention to the relict neutrinos [6] (under which we shall imply antineutrinos, too) filling our Universe. The effective temperature $T \approx 1.9\ K \approx 1.64 \cdot 10^{-13}\ GeV$ of the relict neutrinos is the fairly low one so that it is fulfilled one of conditions ($\lambda \gg r_w$, where λ is the de Broglie's wave-length of a neutrino and r_w is the weak interaction radius of an one [4]) of the propagation of the spin wave in the polarized gases. As a result the quantum effects become the determing ones in such medium and the interference of the neutrinos fields (being the consequence of the known identity of elementary particles) must induce the quantum beats, which will be interpreted as zero oscillations of a vacuum. In consequence of this the mathematical apparatus [7] applied by the description of the Casimir's effect [8] can be used.

We shall be interesting in quantum beats arising by the interference of the falling polarized flow of the relict neutrinos on the macroscopic body with the scattered one at this body. Let's suppose for this the neutrinos have the zero rest mass (the other version [6] will not be considered), so that the direction of their spin is connected hardly with the direction of their 3-velocity. In consequence of this only those neutrinos can be considered as ones forming the polarized flow, which propagate along straight line connecting specifically two particles of different macroscopic bodies. It explain the anisotropy of the zero quantum oscillations, which is necessary to obtain the right dependence ($1/R$) of the energy of the two-particles interaction on the distance R between particles in the Casimir's effect.

Let's consider two macroscopic bodies with masses m_1 and m_2 and with the fairly long distance R one from another. We shall regard, that the bodies contain $2m_1l$ and $2m_2l$ particles correspondingly (where the normalizing factor l is connected with cross-section σ of the neutrino upon the particle), implying thereby the statistics averaging of the properties of the elementary particles constituting the bodies. If the particles of the macroscopic bodies had interacted with all neutrinos incidenting on them then these particles might have been considered as the opaque boandaries, which induce Casimir's effect on the straight line. By this the energy of the interaction of the particles would have been equal to [7]

$$\varepsilon_{AB} = \frac{1}{2}\sum_{n=1}^{\infty}\frac{\pi n}{R_{AB}} - \frac{1}{2}\int_0^{\infty}\frac{\pi x}{R_{AB}}dx =$$

$$= \frac{i}{2}\int_0^{\infty}\frac{\pi(it)/R_{AB} - \pi(-it)/R_{AB}}{\exp(2\pi t) - 1}dt = -\frac{\pi}{24R_{AB}} \tag{1.1}$$

(A is a number of a particle of the first macroscopic body and B is a number of a particle of the second body). On account of the weakness of the interaction of neutrinos with particles we are confined to a first approximation, so that the energy E of the interaction of two macroscopic bodies is equal to

$$E \approx \sum_{A=1}^{2m_1l}\sum_{B=1}^{2m_2l}\varepsilon_{AB}. \tag{1.2}$$

Neglecting the dimensions of the bodies in comparison with interval R between them ($R_{AB} \approx R$), we shall have finally

$$E \approx -2m_1 l\ 2m_2 l\ \pi/(24R) = -G_\nu m_1 m_2/R \tag{1.3}$$

where $G_\nu = \pi l^2/6$.

2. The estimate of the constant G_ν

Consider the scattering of the neutrino upon the charge lepton, induced by the exchange of the neutral Z° boson (taking account of the low energy of the relict neutrinos) only. The ammplitude of the process in the lower approximation can be written down as

$$M = 4G_F 2^{-\frac{1}{2}}(\nu_L^+\gamma_4\gamma^i\nu_L)[(-\frac{1}{2}+\xi)e_L^+\gamma_4\gamma_i e_L + \xi e_R^+\gamma_4\gamma_i e_R] =$$

$$= G_F 2^{-\frac{1}{2}}[\nu^+\gamma_4\gamma^i(1-\gamma_5)\nu][(-\frac{1}{2}+\xi)e^+\gamma_4\gamma_i(1-\gamma_5)e+$$

$$+\xi e^+\gamma_4\gamma_i(1+\gamma_5)e], \tag{2.1}$$

in consequence of this the square of the amplitude (spin-average) will take the form

$$< M^2 >= 64G_F^2[(-\frac{1}{2}+\xi)^2(p'\cdot k')(p\cdot k)+$$

$$+\xi^2(p'\cdot k)(p\cdot k') - (-\frac{1}{2}+\xi)\xi m^2(k'\cdot k)]. \tag{2.2}$$

where γ^i, γ_i are the Dirac's matrices, e is a bispinor describing of a charge lepton, p is its original of 4-momentum and p' is the finite 4-momentum of it (m is the rest mass of a charge lepton); ν is bispinor, describing of the neutrino, k is its original 4-momentum and k' is the finite 4-momentun of it; $+$ is the symbol of the Hermitian conjugation; $G_F \approx 1.166\cdot 10^{-5}\ GeV^{-2}$ is Fermi's constant. Here and further

$$\xi = \sin^2\Theta_W, \quad \gamma_5 = -i\gamma_1\gamma_2\gamma_3\gamma_4,$$

$$\psi_L = \frac{1}{2}(I-\gamma_5)\psi, \quad \psi_R = \frac{1}{2}(I+\gamma_5)\psi. \tag{2.3}$$

(I is the unit matrix and Θ_W is the weak angle). By analogy we can get the square of the scattering amplitude of the antineutrino upon the charge lepton as

$$< M^2 >= 64G_F^2[(-\frac{1}{2}+\xi)^2(p'\cdot k)(p\cdot k')+$$

$$\xi^2(p'\cdot k')(p\cdot k) - (-\frac{1}{2}+\xi)\xi m^2(k\cdot k')]. \tag{2.4}$$

As a result the cross-section of the scattering for the neutrino proves to be equaal to the cross-section of the scattering for the antineutrino in the low-energy approximation (the energy of the neutrino $\omega \ll m$) and they are written down as

$$\sigma^Z = 4G_F^2\omega^2(\frac{1}{4} - \frac{\xi}{2} + \xi^2)/\pi. \tag{2.5}$$

Note that σ^Z proves to be minimal for $\xi = 1/4$. As the low energy neutrinos scarcely are able to change the spin direction of the particles of a macroscopic body, their scattering must be accompanied the collision radiation. In consequence of this the cross-section has the form

$$\sigma_\nu = k_\psi \sigma^Z, \tag{2.6}$$

where for the charge leptons the factor $k_\psi = k_e$ depends on the fine structure constant $\alpha \approx 1/137$ only, while for the quarks the factor $k_\psi = k_q$ must depends on the running coupling constant α_s too, which define the collision radiation by gluons.

For the crude estimate of the constant G_ν let us consider the scattering the relict neutrino upon the electron only, supposing that

$$\sigma_\nu = \pi l^2, k_e = \alpha(\pi^2 - \frac{25}{4})/(2\pi). \tag{2.7}$$

Besides substituting the middling

$$< \omega >= \frac{\int\limits_0^\infty \omega^3 d\omega / \left[\exp\left(\frac{\omega}{T}\right) + 1\right]}{\int\limits_0^\infty \omega^2 d\omega / \left[\exp(\frac{\omega}{T}) + 1\right]} \tag{2.8}$$

instead of ω we receive the following value of the constant

$$G_\nu = \sigma_\nu/6 \approx 10^{-38}\ GeV^{-2} \tag{2.9}$$

($< \omega >\approx 3.15T \approx 5.166 \cdot 10^{-13} GeV, \xi \approx 0.23$) which is near to the known value of the gravitational constant G_N [12].

So the gravitational phenomena can be explained by the presence of the collective oscillations in the neutrinos medium. In consequence it might be worthwhile to return to the potential

$$V(R) = \frac{A}{R}e^{-BR}$$

of which Seeliger [9] suggested to substitute the Newton's potential and to note the gravitational potential (it is possible in an any approximation) as

$$V(R) = \frac{1}{R}\sum_{i=1}^{n} A_i e^{-B_i R} \tag{2.10}$$

in the general case where the constants A_i and B_i characterize the different media. By this we can be based on the theory of the strong gravity (see, for example the work [10]). Moreover, having the neutrinos Universe and taking account of the Fermi-Dirac statistics we can recollect about the Saharov's hipothesis [11] using the idea of the metrical vacuum elasticity for the explanation of the gravitational interactions. But the main idea is it now for us what the normal matter (not neutrinos) acts as the Brownians by the help of which it can make the attempt to estimate the statistics characterization of the Universe neutrinos background. In the capacity of one from such indicator we offer to use the particles masses whiches connect with the scattering cross-section of the neutrinos. Note in tie with it, what we can ignore the photon collision radiation by the neutrinos scattering on the hadrons whiches are the quark resonator because of the existence of the additional degree of freedom in comparison with the electron. Exactly the resonance scattering causes to a gain in the hadrons masses by a factor of 10^3 in comparison with the electron mass. (The great spread of the hadrons masses depend on the form of the collective quark oscilations in the hadrons resonator.)

3. The gauge transformations of the Lie local loop

In order to get the more general equations than Einstein's gravitational equations, one make use of the unified covariant gauge formalism [13]. This formalism is differing from the other ones, because gauge fields are being described by geometrical objects of a tensor type but not by the connections, which perform an other role and which may depend on gauge fields only inderectly. Specifically, the application of connections as counterterms [13] is co-ordinating with the relativistic approach, when the physical sence may be attached to a difference of connections only [14]. Besides we need geometrical objects describing the ground state of a matter (a vacuum), which plays a role of the peculiar thermostat, and connections of various fiber bundles suit for this splendidly. While using connections, corresponding to the spaces of the rather complicated geometrical structure, then one may consider that an excitation state differ from a ground state a little. It allows to consider Lagrangians, depending on the lowest powers of covariant derivatives only. As a result equations of gauge fields will be generalization of Maxwell's equations, but one may obtain in the particular case, which is considered as the limiting one, that they had a quasi-einstein form [15] (when all excitations are being freezed out).

Let's assume that Universe had a stage of a development during which CPT-invariance of physical fields are absent. As known [16] this situation must arise by a breakdown of Lorentz invariance of the space-time M_4 in which only the fields with the elemental structure (scalar fields) can take place. In consequence of physical processes unknown for us this stage of a development was completed by a degeneracy of scalar fields that led to their mixing and breaking down on classes of an equivalence. As a result the laws of the quantum statistics connected directly with the exchange-type interactions have started playing the perceptible

role. Note that the exchange-type interactions will not differ from other interactions formally in our approach and they are only a relict of primary ones after a forming of spinor and vector fields. It is a degeneracy that allows to use vector fiber bundles with the base M_4. Specifically, the fields $\Psi(x)$ will be the cross sections of the vector fiber bundle E_{4+N} (a point $x \in M_4$). Using of approximate symmetries by a description of interactions made it possible to unite non-degenerate fields (in a general case) in multiplets or in supermultiplets, in consequence of what a number N (in an abstract theory) is not concretized.

In addition to the physical fields must be described by not one field $\Psi(x)$, but by the class of the equivalence $\{\Psi(x)\}$ in which the relation of the equivalence is determined by the infinitesimal transformations having the form:

$$\Psi \to \Psi + \delta\Psi = \Psi + \delta\omega^a \; T_a \Psi, \tag{3.1}$$

where $a, b, c, d, e = 1, 2, ..., r$; $\delta\omega^a(x)$ are infinitesimal parameters; $T_a(x)$ are $N \times N$ matrices depending on charges of particles being quanta of fields $\Psi(x)$. Since it is impossible to be fully confident that there is a strict border between internal symmetries and external ones, then it is necessary to consider both transformations of fields $\Psi(x)$ and transformations of points x in the form:

$$x^i \to x^i + \delta x^i = x^i + \delta\omega^a \; \xi^i_a(x), \tag{3.2}$$

(x^i are coordinates of a point $x \in M_4$; $i, j, k, l, m, n, p, q = 1, 2, 3, 4$). In consequence of this it might be worthwhile to resolve $\delta\Psi$ into summands as follows

$$\delta\Psi = \delta_0\Psi + \delta\omega^a \; \xi^i_a \; \partial_i \Psi \tag{3.3}$$

($\partial_i\Psi$ are the partial derivatives of fields $\Psi(x)$), selecting the changes $\delta_\circ\Psi$ of fields $\Psi(x)$ in the point x. Writing down $\delta_0\Psi$ as

$$\delta_0\Psi = \delta\omega^a \; X_a(\Psi) = \delta\omega^a \; (T_a\Psi - \xi^i_a \; \partial_i\Psi), \tag{3.4}$$

we shall regard $X_a(\Psi)$ the generators of the Lie local loop G_r. We refuse the associativity property which is inherent to the Lie local groups (see for example the work [17] of M.Kikkawa about a connection of a geometry of a space with a structure of a Lie local loop).

Being on the relativistic positions it is necessary to assume that the change $\delta\Psi$ can be only with the finite rate and they are produced by an exchange of particles or quasiparticles being quanta of the special fields $B(x)$, the connection of which with the fields $\Psi(x)$ (in Lagrangian) must have the form:

$$D_\beta\Psi = -B^a_\beta X_a(\Psi), \tag{3.5}$$

where $B^a_\alpha(x)$ are components of fields $B(x)$ with a consideration of their factorization on the Lie local loop G_r. Note, that $B^a_\alpha(x)$ can be both Utiyama's gauge fields [18] and Kibble's gauge fields [19]. Following for Utiyama [18] we shall not concretize classes of an equivalence

of fields $B(x)$, in consequence of what we shall not concretize significances which are adopted by indexes α, β, γ, δ, ε, θ. Resolving the matrices T_a into summands as

$$T_a = L_a - \xi^i_a \ \Gamma_i \tag{3.6}$$

we write down $X_a(\Psi)$ in the form:

$$X_a(\Psi) = L_a\Psi - \xi^i_a \ \nabla_i\Psi, \tag{3.7}$$

where ∇_i are symbols of covariant derivatives.

Of course the connections Γ_i (as and any different ones) can considered as the gauge fields too, but only if we shall call the coordinate transformations as the gauge ones, which (in our opinion) must characterize the space of the observer with his instruments (containing the primary standards) and also the his method of the description (the physical model and the mathematical formalism).

Let the type of geometrical objects to be conserved by the transformations of the Lie local loop G_r for what it is enough to demand that $L_a(x)$ and $\xi^i_a(x)$ are the linear homogeneous geometrical objects satisfying to the following relations:

$$\xi^i_a \ \nabla_i\xi^k_b - \xi^i_b \ \nabla_i\xi^k_a - 2 \ S^k_{ij} \ \xi^i_a \ \xi^j_b = -C^c_{ab} \ \xi^k_c, \tag{3.8}$$

$$L_aL_b - L_bL_a - \xi^i_a \ \nabla_iL_a + \xi^i_b \ \nabla_iL_a + R_{ij} \ \xi^i_a \ \xi^j_b = C^c_{ab} \ L_c, \tag{3.9}$$

where $S^k_{ij}(x)$ are the components of the torsion of the space-time M_4

$$S^k_{ij} = (\Gamma^k_{ij} - \Gamma^k_{ji})/2 \tag{3.10}$$

and $R_{ij}(x)$ are the components of the curvature of the connection $\Gamma_i(x)$

$$R_{ij} = \partial_i\Gamma_j - \partial_j\Gamma_i + \Gamma_i\Gamma_j - \Gamma_j\Gamma_i. \tag{3.11}$$

The components $C^c_{ab}(x)$ of the structural tensor of the Lie local loop G_r must satisfy to the identities:

$$C^c_{ab} + C^c_{ba} = 0, \tag{3.12}$$

$$C^d_{[ab} \ C^e_{c]d} + \xi^i_{[a} \ \nabla_{|i|}C^e_{bc]} - R_{ij[a}{}^e \ \xi^i_b \ \xi^j_{c]} = 0, \tag{3.13}$$

where $R_{ija}{}^e(x)$ are the components of the curvature of the connection $\Gamma_{ia}{}^b(x)$

$$R_{ijb}{}^a = \partial_i\Gamma_{jb}{}^a - \partial_j\Gamma_{ib}{}^a + \Gamma_{ic}{}^a\Gamma_{jb}{}^c - \Gamma_{jc}{}^a\Gamma_{ib}{}^c. \tag{3.14}$$

4. The gauge fields

Now one may proceed to a construction of the covariant gauge formalism following to R.Utiyama's classical work [18]. For this it is necessary to find a law of a transformation of the fields $B(x)$. Let the fields $D_\alpha\Psi$ change analogously to the fields $\Psi(x)$ in a point $x \in M_4$, then is

$$\delta_0 D_\alpha \Psi = \delta\omega^b \; (L_b D_\alpha \Psi - D_\beta \Psi \; L_{b\alpha}{}^\beta - \xi^i_b \; \nabla_i D_\alpha \Psi), \tag{4.1}$$

where the components $L_{a\alpha}{}^\beta(x)$ of linear homogeneous geometrical objects satisfy the following relations:

$$L_{d\beta}{}^\gamma \; L_{b\alpha}{}^\beta - L_{b\beta}{}^\gamma \; L_{d\alpha}{}^\beta - \xi^i_d \; \nabla_i L_{b\alpha}{}^\gamma + \xi^i_b \; \nabla_i L_{d\alpha}{}^\gamma + R_{ij\alpha}{}^\gamma \; \xi^i_d \; \xi^j_b = C^c_{db} \; L_{c\alpha}{}^\gamma \tag{4.2}$$

($R_{ij\alpha}{}^\gamma(x)$ are the components of the curvature of the connection $\Gamma_{i\alpha}{}^\gamma(x)$). As a result $\delta_0 B^a_\alpha$ are written down in the form:

$$\delta_0 B^d_\alpha = \delta\omega^b \; (C^d_{cb} \; B^c_\alpha - B^d_\beta \; L_{b\alpha}{}^\beta - \xi^i_b \; \nabla_i B^d_\alpha) + \Phi^i_\alpha \; (\nabla_i \delta\omega^d - V_i \; \delta\omega^d), \tag{4.3}$$

where

$$\Phi^i_\beta = B^a_\beta \; \xi^i_a, \tag{4.4}$$

$$V_i = \Gamma_{ij}{}^j - \Gamma^o{}_{ij}{}^j. \tag{4.5}$$

Here and further $\Gamma_{ij}{}^k(x)$ are the components of the internal connection of the space-time M_4 and $\Gamma^o{}_{ij}{}^k(x)$ are the components of the connection of an equiaffine space of an affine connection being a locally diffeomorphic one to the space of an affine connection M_4. As $\delta_0 B^c_\alpha$ depend on $\nabla_i \delta\omega^a$ then $B(x)$ are called the gauge fields.

Since the action

$$\int\limits_{\Omega_4} \mathcal{L} \eta dx^1 dx^2 dx^3 dx^4 \tag{4.6}$$

(Ω_4 is a region of the space-time M_4 and $\eta(x)$ is the base density of the same) must be invariant against infinitesimal transformations of the Lie local loop G_r, then the total Lagrangian $\mathcal{L}$ depending on fields $\Psi(x)$, $B(x)$ and also their derivatives of the first order is unable to be selected arbitrarily. The following Lagrangian $\mathcal{L}(\Psi; D_\alpha\Psi; F^c_{\alpha\beta})$ satisfy to this demand, where the components $F^c_{\alpha\beta}(x)$ of the intensities of the gauge fields $B(x)$ have the form:

$$F^c_{\alpha\beta} = [\delta^c_b - \xi^i_b \; \Phi^\gamma_i \; (B^c_\gamma - \beta^c_\gamma)] \; [\Phi^j_\alpha \; \nabla_j B^b_\beta - \Phi^j_\beta \; \nabla_j B^b_\alpha - B^e_\alpha \; B^d_\beta \; C^b_{ed} + (B^e_\alpha \; L_{e\beta}{}^\delta - B^e_\beta \; L_{e\alpha}{}^\delta) \; B^b_\delta]. \tag{4.7}$$

Note that the fields $\Phi^\alpha_i(x)$ are defined from the equations: $\Phi^i_\alpha \; \Phi^\alpha_j = \delta^i_j$ (δ^j_i and δ^b_a are the Kronecker delta symbols). The components $\beta^b_\alpha(x)$ of linear homogeneous geometrical objects are being interpreted as vacuum averages of gauge fields $B(x)$.

Rewrite the equations

$$\Phi^i_\alpha \left(\frac{\mathcal{L}}{\eta} \frac{\partial \eta}{\partial B^b_\alpha} + \frac{\partial \mathcal{L}}{\partial B^b_\alpha} - \nabla_j \left(\frac{\partial \mathcal{L}}{\partial \nabla_j B^b_\alpha} \right) \right) = 0 \tag{4.8}$$

of gauge fields in the quasi-maxwell form:

$$\nabla_j H^{ji}_a - H^{jk}_a \; S^i_{jk} = I^i_a, \tag{4.9}$$

where

$$H^{ij}_a = -\Phi^i_\beta \; \frac{\partial \mathcal{L}}{\partial \nabla_j B^a_\beta} = \Phi^j_\beta \; \frac{\partial \mathcal{L}}{\partial \nabla_i B^a_\beta}, \tag{4.10}$$

$$I^i_a = -\mathcal{L}\xi^i_a - \frac{\partial \mathcal{L}}{\partial \nabla_i \Psi} \; X_a(\Psi) - \frac{\partial \mathcal{L}}{\partial \nabla_i B^b_\beta} \; Y^b_{a\beta}(B), \tag{4.11}$$

$$Y^b_{a\gamma}(B) = C^b_{ca} \; B^c_\gamma - B^b_\beta \; L^\beta_{a\gamma} - \xi^i_a \; \nabla_i B^b_\gamma. \tag{4.12}$$

We pick out from the equations of gauge fields folding them with $B^b_\alpha \; \Phi^\alpha_l$ those which can will be called the equations of fields $\Phi^i_\alpha(x)$ and which must substitute for Einstein's gravitational equations in a general case. When all excitation will be freezed out ($B^b_\alpha \to \beta^b_\alpha$) these equations must transfer to equations of a ground state of a matter (a vacuum), it is desirable to write down in the geometrized form (a quasi-einstein form) for the conservation of the Einstein's ideology.

5. The generalized Einstein's equations

Consider the specifical case, when Lagrangian has the following form

$$\mathcal{L}_t = F^a_{\alpha\beta} F^b_{\gamma\delta} \eta^{\beta\delta} [\kappa \xi^i_a \xi^j_b (\eta^{\alpha\gamma} \eta_{\varepsilon\theta} h^\varepsilon_i h^\theta_j + 2 h^\gamma_i h^\alpha_j - 4 h^\alpha_i h^\gamma_j) +$$

$$\kappa_1 \eta_{cd} \eta^{\alpha\gamma} (\delta^c_a - \xi^i_a h^\varepsilon_i \beta^c_\varepsilon)(\delta^d_b - \xi^j_b h^\theta_j \beta^d_\theta)]/4 + \mathcal{L}(\Psi, D_\alpha \Psi), \tag{5.1}$$

where κ and κ_1 are the constant ones; $\alpha, \beta, \gamma, \delta, \varepsilon, \theta = 1, 2, 3, 4$; $\eta_{ab}(x)$, $\eta_{\alpha\beta}$ and $\eta^{\gamma\delta}$ are the components of the symmetric undegenerate tensor fields, by this $\eta_{\alpha\beta}\eta^{\gamma\beta} = \delta^\gamma_\alpha$. Besides the fields $h^i_\alpha(x)$ are defined in the form

$$h^i_\alpha = \beta^a_\alpha \xi^i_a \tag{5.2}$$

and the fields h^α_i are defined from the following equations $h^\alpha_i h^j_\alpha = \delta^j_i$. Moreover let

$$g^{ij} = \eta^{\alpha\beta} \Phi^i_\alpha \Phi^j_\beta, \tag{5.3}$$

$$L_{a\alpha}{}^\beta = \xi^i_a L_{i\alpha}{}^\beta, \quad L_{k\gamma}{}^\alpha \eta^{\gamma\beta} + L_{k\gamma}{}^\beta \eta^{\gamma\alpha} = \eta^{\alpha\beta} l_k, \tag{5.4}$$

where

$$l_k = (4 S^i_{ki} + g^{ij} \nabla_k g_{ij})/2. \tag{5.5}$$

By this the fields g_{ij} are defined from the correlations $g_{ij}g^{kj} = \delta^k_i$. As a result it can receive the equations

$$g^{jl}R_{ikl}{}^i - \frac{1}{2}\delta^j_k g^{ml}R_{iml}{}^i +$$
$$\frac{1}{2}\nabla_i(2g_{kp}g^{il}g^{jq}S^p_{ql} - g_{lk}g^{jq}\nabla_q g^{il} + g_{kq}g^{im}\nabla_m g^{qj} + S^j_{il}g^{im}g^{ln}(\nabla_m g_{kn} + S^p_{mn}g_{kp}) +$$
$$g^{ij}[\frac{1}{2}(\nabla_k l_i - \nabla_i l_k) - l_l S^l_{ik} - \frac{1}{2}\nabla_m(g_{in}\nabla_k g^{mn})\nabla_m(g^{ml}S^n_{kl}g_{in}) + \nabla_m S^m_{ik} +$$
$$\frac{1}{4}\nabla_k g^{lm}\nabla_i g_{lm} + S^l_{mk}g^{mn}\nabla_i g_{ln} - \frac{1}{4}l_k(4S^l_{li} + g_{lm}\nabla_i g^{lm}) - S^n_{pi}g_{mn}\nabla_k g^{mp} -$$
$$2S^l_{mk}(S^m_{li} + g_{ln}g^{mp}S^n_{pi})] + \frac{1}{8}\delta^j_k[4\nabla_i\nabla_l g^{il} - 2g_{mn}\nabla_i g^{lm}\nabla_l g^{in} - g^{il}\nabla_i g^{mn}\nabla_l g_{mn} +$$
$$l^i(4S^l_{li} + g_{lm}\nabla_i g^{lm}) + 4S^m_{il}g^{ln}(2S^i_{mn} + g_{mp}g^{iq}S^p_{qn}) - 8S^n_{pq}g^{pm}g^{qi}\nabla_i g_{mn}] =$$
$$\frac{1}{2\kappa}[D^{ij}_a E^a_{ik} - \frac{1}{4}\delta^j_k D^{il}_a E^a_{il} + P^j\Psi D_k\Psi - \delta^j_k\mathcal{L}(\Psi, D_i\Psi)], \tag{5.6}$$

where

$$D_i\Psi = \Phi^\alpha_i D_\alpha\Psi = \nabla_i\Psi - B^a_i L_a\Psi, \tag{5.7}$$
$$P^k\Psi = \frac{\partial\mathcal{L}}{\partial D_k\Psi} = \Phi^k_\alpha\frac{\partial\mathcal{L}}{\partial D_\alpha\Psi}, \tag{5.8}$$
$$B^a_i = \Phi^\alpha_i B^a_\alpha, \tag{5.9}$$
$$E^a_{ij} = (\delta^a_b - \xi^k_b B^a_k)(\nabla_i B^b_j - \nabla_j B^b_i + B^c_i B^d_j C^b_{cd}), \tag{5.10}$$
$$D^{ij}_a = \kappa_1 g^{ik}g^{jl}g_{ab}E^b_{kl}, \tag{5.11}$$

which can be used for the description of both the medium with the amply wide spectrum of properties and the unsterile vacuum “spoiled” by the presence of fields.

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